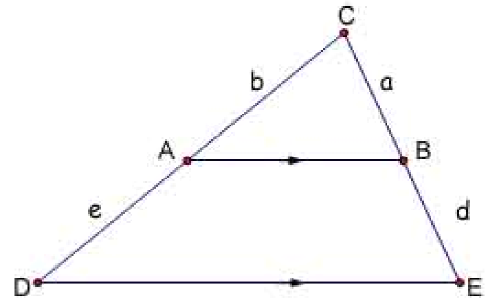
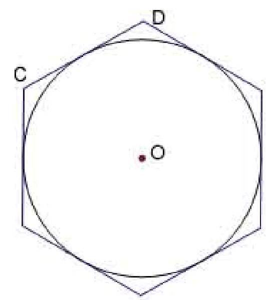
Math 5 – Some Problems to study for the final exam

1. Refer to the diagram to the right, where $AB \parallel DE$.
   a. Prove that $\Delta CAB \sim \Delta CDE$
   b. Prove that $\frac{a}{b} = \frac{d}{e}$.
   c. Use this result to prove that the base angles of an isosceles trapezoid are congruent.



2. Consider the circle inscribed in a regular hexagon, as shown in the diagram to the right. Let CD = 8,

   a. What is the measure of central angle $\angle COD$?
   b. What is the area of the circle?|
   c. What is the area of the hexagon?
   d. What is the difference between the area of the circle and the area of the hexagon.



3. Find all values of t so that the distance from the origin to the point with coordinates $(t - 1, t^2 - 2)$ is the same as the distance from the origin to the point with coordinates $(3, t)$.

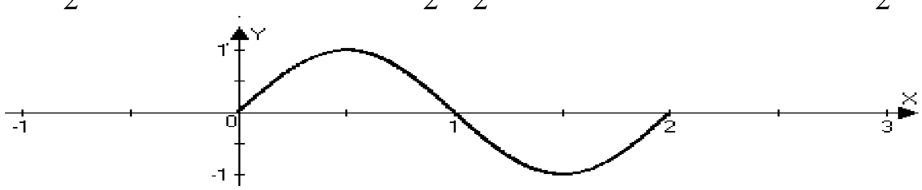
4. Consider the function $f(x) = x^2$. Find and simplify an expression for the slope of the line connecting $(x, f(x))$ with $(x^2, f(x^2))$. Find the minimum value of this slope.

5. Compute and simplify the average rate of change of $f(x) = \frac{x}{x^2 + 1}$ over the given interval. Simplify.
   a. $[0, 2]$ b. $[1, 1+h]$

6. Consider the quadratic $f(x) = -x^2 + 2x + 2$
   a. Express the quadratic function in vertex form.
   b. Express the zeros (x-intercepts) of the parabola in simplest radical form.

7. Sketch its graph, showing the coordinates of the vertex and all intercepts.

8. The graph of a function g is given below. Use it to tabulate 5 key points and graph the following:
   a. $y = \frac{1}{2} g(2x)$ b. $y = \frac{1}{2} + \frac{1}{2} g(2x+1)$ c. $y = -\frac{1}{2} - \frac{1}{2} g(2x+1)$



9. A child rides on a merry-go-round horse which is at a distance of 15 ft from the center of the merry-go-round. After 8 minutes the child had traveled 13.4 revolutions.
   a. What is the radian measure of the angle the child has swept out during the 8 minutes?
   b. What linear distance has the child traveled?
   c. What was the average angular speed of the child during the 8 minutes?

10. Suppose tan $\theta = 24/7$ and θ is in the third quadrant. Find the following:

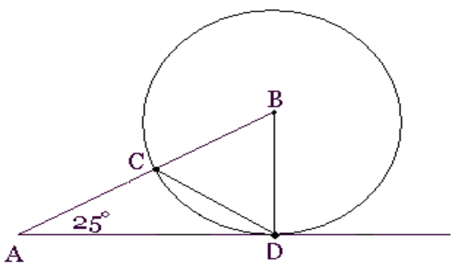
   a. $sin\theta$ b. $\cos(\theta + \frac{\pi}{2})$ c. $\cos\ (\theta + \frac{3\pi}{2})$

11. The Millennium Wheel rotates once every 15 minutes. Its highest point is about 142 meters above the ground and the lowest point is about 2 meters above the ground. Write a function that gives the height of a rider *t* minutes after boarding the Millennium Wheel.

12. The total surface area of a cylinder is π square units.
   a. Find a function that models the cylinder's height as a function of its radius.
   b. Find a function that models the cylinder's radius as a function of its height.

13. Find a formula for the inverse function of $f(x) = \sqrt[3]{x+8}$ and plot the function and its inverse together in the same coordinate plane, showing the symmetry of these function across the line $y = x$.

14. In the figure at right, suppose the circle centered at *B* has radius 4.
   a. What is the length of *AD*?
   b. What is the length of *AC*?
   c. What is a measure of angle $\angle ACD$?
   d. What is the length of *CD*?



15. Suppose that a triangle has sides of length 4 and 6 which include the angle $\theta = 1$. Solve the triangle.

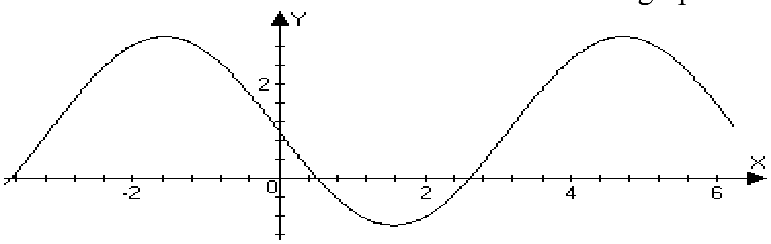
16. Suppose that a triangle has sides with lengths 3, 8 and 10. Find the interior angles.

17. Suppose $\cos\theta = -\frac{3}{7}$ in quadrant III. Find $\csc\theta$, in simplest radical form.

18. Suppose a point in the fourth quadrant of the unit circle has *x* coordinate $\frac{1}{5}$.
   a. Draw the unit circle in the Cartesian plane showing the position of this point.
   b. Find the radian measure of the smallest positive polar angle θ that terminates at this point.
   c. What are the coordinates of the terminal point for $\theta - \pi$?
   d. What are the coordinates of the terminal point for $\theta - \frac{\pi}{2}$?

19. Consider $f(x) = 15 + 30\sin\left(\frac{\pi}{12}x + \frac{\pi}{8}\right)$
   a. Find the amplitude, period, phase shift and equilibrium line for *f*.
   b. Sketch a graph for $f(x) = 15 + 30\sin\left(\frac{\pi}{12}x + \frac{\pi}{8}\right)$ labeling the coordinates of points at the max, min and equilibrium points.
   c. Find all values of *x* such that $15 + 30\sin\left(\frac{\pi}{12}x + \frac{\pi}{8}\right) = 0$. Write the values in exact form.

20. Find a formula for the sinusoidal function whose graph is shown:



21. A Ferris wheel of radius 18 meters is positioned so that its center is 31 meters above the ground. Write a sinusoidal function which gives the height of a point which starts at the bottom of the Ferris wheel, assuming that the wheel turns 2 revolutions every 3 minutes.

22. Given $\sin\theta = \frac{7}{25}$, and θ is in the second quadrant, simplify expressions for $\cos\theta$ and $\tan\theta$.

Do not approximate these, but give approximate 4-digit approximation for both the radian and degree measure of θ.

23. Given $\cos(t) = \frac{1}{4}$, and t is in the fourth quadrant, simplify expressions for

a. $\cos(t+\pi)$ b. $\cos(t+\frac{\pi}{2})$ c. $\tan(t+\frac{\pi}{2})$

24. Solve the triangle given

a. Three sides with lengths 34, 56 and 89.

b. Angle $\angle A = 30^\circ$ opposite $a = 17$ and adjacent to $c = 8$.

25. Consider the function $f(x) = 1 - \sqrt{2}\sin\left(2x - \frac{\pi}{3}\right)$

a. Find the amplitude, period and phase shift for and sketch a graph showing at least one wave form. Be careful scale and label axes in your graphs.

b. Find all x-intercepts of $f(x) = 1 - \sqrt{2}\sin\left(2x - \frac{\pi}{3}\right)$. Write the values in exact form.

26. A disk centered at the origin with radius 4000 miles is rotating at a rate of 1 revolution every 24 hours. Consider a point P which starts on the x-axis with coordinates (4000, 0). We're interested in the distance from P to the nearest point on the vertical line $x = 24000$ miles.

a. Sketch a LARGE graph for this situation. Scale and label significant points and lengths.

b. Compute the angular velocity of the disk, in radians per hour.

c. Write a sinusoidal function which gives this distance (from P to the vertical line) as a function of t, measured in hours.

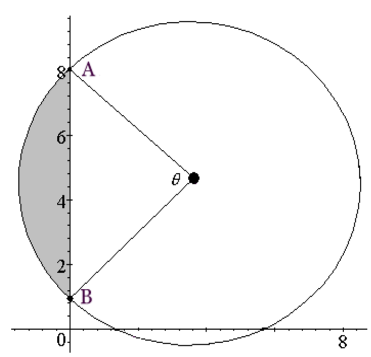
27. The circle shown at right has the equation $\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = 25$.

a. Find the distance between the y-intercepts.

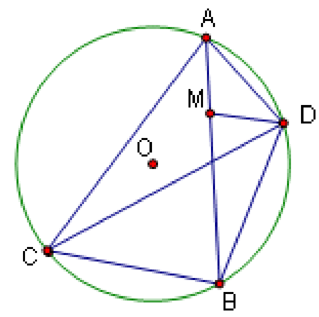
b. Show that the central angle θ of the sector is given by

$$\theta = 2\arcsin\left(\frac{\sqrt{51}}{10}\right).$$

c. Approximate the shaded area to the nearest thousandth.



28. The diagram shows circular quadrilateral *ABCD* with diagonals AB and CD.
   a. Show that if we construct M as shown so that $\angle ADM \cong \angle CDB$ then $AD \cdot CB = AM \cdot CD$
   b. Show that $AC \cdot BD = BM \cdot CD$
   c. Conclude that $CD \cdot AB = AC \cdot BD + AD \cdot CB$



29. Find the area of triangle with sides of lengths *a*, *b*, and *c* opposite angles A, B, and C.
   a. In terms of *a* and *b* and the included angle.
   b. In terms of *a* and *c* and the included angle.
   c. In terms of *b* and *c* and the included angle.
   d. Use the fact that all these measures of the area are equal to prove the law of sines.

30. Find an equation for the hyperbola with asymptotes $y - 2 = \pm 2(x - 3)$ and foci $(3 \pm \sqrt{5}, 2)$.

31. Find an equation for the ellipse centered at (3,4) with eccentricity ½ and passing through (0,0).

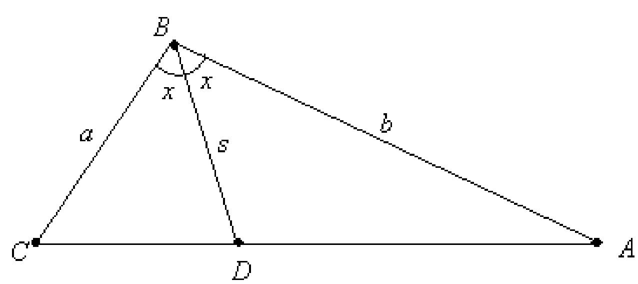
32. Parameterize the given conic section using trigonometric functions. Sketch a graph showing the key features of the conic.

a. $x^2 = 4 - 2y^2$ b. $\dfrac{(x-1)^2}{3} = 2(y-4)^2 = 1$ c. $5x^2 - 4x - 3y^2 + 2y + 1 = 0$

33. Suppose angle C = 10° in a triangle ABC is formed by adjacent sides of length *a* = 3 and *b* =5. What is the length of the opposite side, *c*?

34. Express $\sec\left(\arctan\frac{1}{2}\right)$ in simplest radical form.

35. In the diagram at right, *BD* bisects angle *B* and has length *s*. Use the idea that areas Δ*BCD* + area Δ*BAD* = area of Δ*ABC* to show that $s = \dfrac{2ab\cos x}{a+b}$



36. Rewrite the expression as an algebraic expression in *x*: $\sin(\arctan x)$.

37. Given that $f(x) = \arctan x$, sketch a graph for $y = 1 + 2f(\pi(x+2))$ and find asymptotes and intercepts.